Journal of Engineering Physics and Thermophysics, Vol. 78, No. 3, 2005

INFLUENCE OF THE PIEZOELECTRIC EFFECT ON THE PROPAGATION OF ELASTIC WAVES IN CUBICALLY ANISOTROPIC PIEZOELECTRICS

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UDC 539.3

Using bismuth germanate as an example, it has been shown that longitudinal and quasitransverse waves propagating in the $(1\bar{1}0)$ *crystallographic plane of a cubically anisotropic medium are piezoelectrically active.*

Reciprocal-velocity surfaces and three-dimensional wave fronts for quasilongitudinal and quasitransverse waves propagating in cubically anisotropic piezoelectric materials have been constructed in [1]. In the present work, we give results of investigations of the regularities of propagation of elastic and piezoactive waves in the coordinate plane $x_1' = 0$ of the (x_1', x_2', x_3') system described in [2]. According to [1], the characteristic equation will be written in the following form:

$$q_0 p_0^6 / c_2^6 + q_1 p_0^4 / c_2^4 + q_2 p_0^2 / c_2^2 + q_3 = 0 \,, \tag{1}$$

where

$$q_0 = -1 \,; \quad q_1 = (2 + a)\,\tau_1 + \frac{K\tau_2}{\tau_1} \,;$$

$$q_2 = ((a + b)\,(b - a + 2) - 2K)\,\tau_2 - (1 + 2a)\,\tau_1^2 - \frac{K}{\tau_1}\,(a - 1)\,(\tau_1\tau_2 - 3\tau_3) + \frac{6\,(b + 1)\,K\tau_3}{\tau_1} \,;$$

$$\begin{aligned} q_3 = a\tau_1^3 + (a - 1)\,K\,(\tau_1\tau_2 - 3\tau_3) + \frac{K}{\tau_1}\,(a + b)\,(a - b - 2) \times \\ \times (\tau_2^2 - 2\tau_1\tau_2) + ((a + b)\,(a - b - 2) + K)\,\tau_1\tau_2 + \\ + ((a - b - 2)^2\,(a + 2b + 1) - 2\,(1 + b)\,(a + 1 - b)\,K)\,\tau_3 \,. \end{aligned} \tag{2}$$

We divide both sides of (1) by g^6 $(g = \sqrt{p_1^2 + p_2^2 + p_3^2} \neq 0)$. As a result, we will have

$$q_0 v^6 + \tilde{q}_1 v^4 + \tilde{q}_2 v^2 + \tilde{q}_3 = 0 \,. \tag{3}$$

Here the formulas for $\tilde{q}_i$ follow from (2) upon the replacement of τ_i by $\tilde{\tau}_i$, $i = \overline{1, 3}$, in them.

The solution of (3) will be represented in the following form:

$$v_m = \sqrt{2\sqrt{-\frac{\tilde{p}}{3}}\cos\left(\frac{\tilde{\Lambda} + 2\pi\,(4 - m)}{3}\right) - \frac{\tilde{q}_1}{3q_0}} \,, \quad \tilde{\Lambda} = \arccos\left(-\frac{\tilde{q}}{2}\sqrt{-\left(\frac{3}{\tilde{p}}\right)^3}\right), \tag{4}$$

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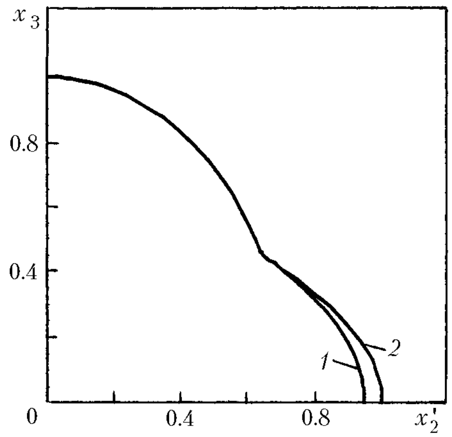

Fig. 1. Surfaces of reciprocal velocities $1/v_3$ in the plane $x_1' = 0$ for bismuth germanate with allowance for the piezoeffect (1) and without allowance for it (2).

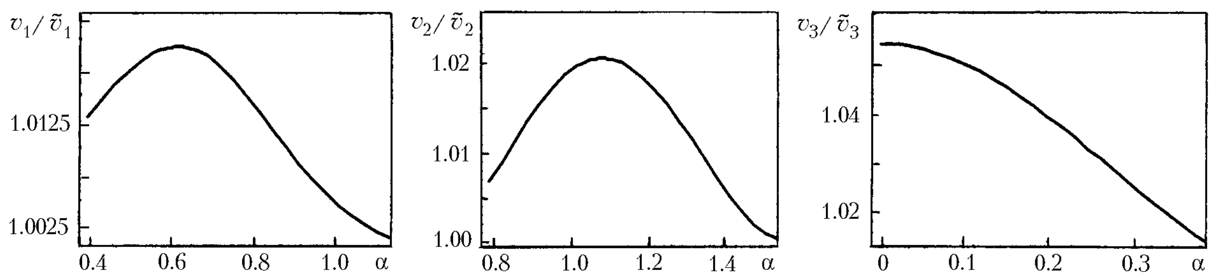

Fig. 2. Ratios $v_m/\tilde{v}_m$ vs. angle of inclination of the normal to the characteristic surface α for quasilongitudinal and quasitransverse waves propagating in the plane $x_0' = 0$ for bismuth germanate.

where $\tilde{p} = -\dfrac{\tilde{q}_1^2}{3q_0^2} + \dfrac{\tilde{q}_2}{q_0}$; $\tilde{q} = \dfrac{2\tilde{q}_1^3}{27q_0^3} - \dfrac{\tilde{q}_1\tilde{q}_2}{3q_0^2} + \dfrac{\tilde{q}_3}{q_0}$; $m = \overline{1, 3}$.

Formulas (4) enable us to construct dimensionless reciprocal-velocity surfaces and their sections by the planes passing through the origin of coordinates. A comparative analysis of the reciprocal-velocity surfaces shows that $v_1 > v_2 \geq v_3$; therefore, we will assume [3] that a quasilongitudinal wave propagates with a velocity v_1, whereas quasitransverse waves propagate with velocities v_2 and v_3. Figure 1 gives the section of the surface of reciprocal velocities $1/v_3$ by the coordinate plane $x_1' = 0$ of the coordinate system (x_1', x_2', x_3') for bismuth germanate ($a = 5.02$, $b = 1.12$, and $K = 0.45$; the numerical data have been taken from [3]).

From Fig. 1 it is clear that the quasitransverse wave propagating with a velocity v_3 is piezoactive in the plane $x_1' = 0$. The interval between curves 1 and 2 on the x_2' axis coincides with the largest distance between the reciprocal-velocity curves in the plane $x_3 = 0$, which have been constructed in [2] for the case of plane deformation of a cubically anisotropic medium. The absence of the piezoactivity of quasilongitudinal and quasitransverse waves in the coordinate plane $x_3 = 0$ has also been noted in [2]. However, the sections of the surfaces of reciprocal velocities $1/v_1$ and $1/v_2$ by non-coordinate planes show that the piezoelectric effect exerts an influence on the change in the velocities v_m of propagation of these types of waves. Figure 2 gives the velocity ratios $v_m/\tilde{v}_m$ of waves propagating in the plane $x_1' = 0$ of bismuth germanate as functions of the angle of inclination of the normal to the characteristic surface ($\tilde{v}_m$ are the velocities of propagation of elastic waves without allowance for the piezoeffect).

We find the coordinates of wave-front points reached by the energy of wave disturbance by the instant of time t. For this purpose we express p_0 from Eq. (1):

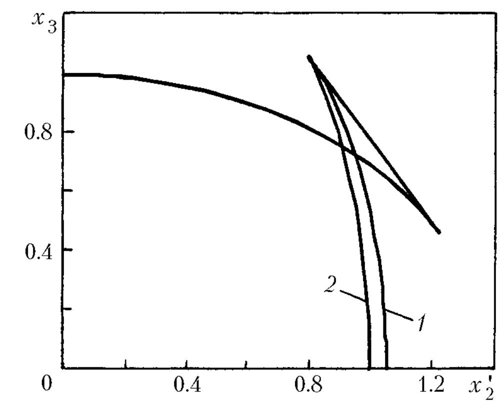

Fig. 3. Front of the quasitransverse wave propagating with a velocity v_2 in the plane $x_1' = 0$ for bismuth germanate with allowance for the piezoeffect (1) and without allowance for it (2).

$$p_0^{(m)} = c_2 \sqrt{2\sqrt{-\frac{p}{3}}\cos\left(\frac{\Lambda + 2\pi m}{3}\right) - \frac{q_1}{3q_0}}\,, \quad \Lambda = \arccos\left(-\frac{q}{2}\sqrt{-\left(\frac{3}{p}\right)^3}\right), \tag{5}$$

where $p = -\frac{q_1^2}{3q_0^2} + \frac{q_2}{q_0}$ and $q = \frac{2q_1^3}{27q_0^3} - \frac{q_1 q_2}{3q_0^2} + \frac{q_3}{q_0}$; the superscript $m = \overline{1, 3}$ points to the type of elastic wave.

Differentiating p_0 with respect to p_i and integrating the resulting expressions with respect to the time t, we obtain the expressions sought for the dimensionless coordinates:

$$\frac{x_i^{(m)}}{c_2 t} = \frac{1}{v_i^{(m)}}\left(\frac{1}{2\sqrt{-3p}}\left(\frac{2q_1 q_{1i}}{3q_0^2} - \frac{q_{2i}}{q_0}\right)\cos\left(\frac{\Lambda + 2\pi m}{3}\right) - \frac{1}{3}\sqrt{-\frac{p}{3}}\,\times\right.$$
$$\times \sin\left(\frac{\Lambda + 2\pi m}{3}\right)\sqrt{\frac{p^3}{4p^3 + 27q^2}}\left(\left(\frac{2q_1^2 q_{1i}}{9q_0^3} - \frac{q_2 q_{1i} + q_1 q_{2i}}{3q_0^2} + \frac{q_{3i}}{q_0}\right)\times\right.$$
$$\left.\left.\times \sqrt{\left(-\frac{3}{p}\right)^3} - \frac{9q\sqrt{3}}{2}\sqrt{\left(-\frac{1}{p}\right)^5}\left(\frac{2q_1 q_{1i}}{3q_0^2} - \frac{q_{2i}}{q_0}\right)\right)\right). \tag{6}$$

The superscript m in formulas (6) points to the type of wave: 1 corresponds to a quasilongitudinal wave and 2 and 3 correspond to quasitransverse waves.

The coefficients q_{ki}, $k = \overline{1, 3}$, with account for $p_k = gn_k$ will be represented in the form

$$q_{1i} = (2 + a)\,\tilde{\tau}_{1i} + \frac{K(\tau_{2i}\tilde{\tau}_1 - \tau_{1i}\tilde{\tau}_2)}{\tilde{\tau}_1^2}\,,$$

$$q_{2i} = ((a + b)(b - a + 2) - 2K)\,\tau_{2i} - 2(1 + 2a)\,\tilde{\tau}_1 \tau_{1i} - \frac{6K(b + 1)(a - 1)}{\tilde{\tau}_1^2}\times$$

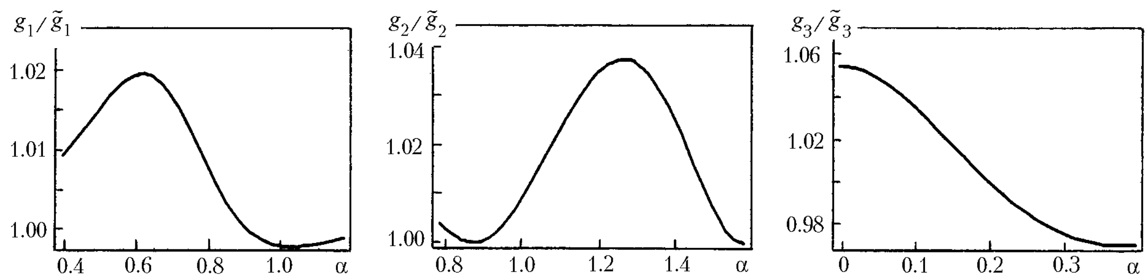

Fig. 4. Ratios of the ray velocities $g_m/\tilde{g}_m$ vs. angle of inclination of the normal to the characteristic surface α for the quasilongitudinal and quasitransverse waves propagating in the plane $x_1' = 0$.

$$\times ((\tau_{1i}\tilde{\tau}_2 + \tilde{\tau}_1\tau_{2i} - 3\tau_{3i})\,\tilde{\tau}_1 - (\tilde{\tau}_1\tilde{\tau}_2 - 3\tilde{\tau}_3)\,\tau_{1i}) - \frac{6K\,(b+1)\,(\tilde{\tau}_3\tau_{1i} - \tau_{3i}\tilde{\tau}_1)}{\tilde{\tau}_1^2}\,,$$

$$q_{3i} = 3a\tilde{\tau}_1^2\,\tau_{1i} + ((a-b-2)\,(a+b) + K)\,(\tau_{1i}\tilde{\tau}_2 + \tilde{\tau}_1\tau_{2i}) + (a-1)\,K \times$$

$$\times (\tau_{1i}\tilde{\tau}_2 + \tilde{\tau}_1\tau_{2i} - 3\tau_{3i}) + ((a-b-2)^2\,(a+2b+1) - 2\,(1+b)\,(a+1-b)\,K)\,\tau_{3i} +$$

$$+ \frac{2K}{\tau_1^2}\,(a+b)\,(a-b-2)\,(\tau_1\,(\tau_2\tau_{2i} - (\tau_{1i}\tau_3 + \tau_1\tau_{3i})) - \tau_{1i}\,(\tau_2^2 - 2\tau_1\tau_3))\,,$$

where $\tau_{1i} = 2\pi_i$, $\tau_{2i} = 2n_i(\tau_1 - n_i^2)$, and $\tau_{3i} = 2n_i(\tau_2 - n_i^2(\tau_1 - n_i^2))$.

Figure 3 shows the section of the front of a quasitransverse wave propagating with a velocity v_3 by the plane $x_1' = 0$ for bismuth germanate with allowance for the piezoeffect and without allowance for it at the instant of time $t = 1$ sec.

It is seen that the quasitransverse wave propagating with a velocity v_3 is piezoactive in the plane $x_1' = 0$. The influence of the piezoelectric effect leads to a decrease of 4^{o} in the lacuna angle (by the lacuna angle we mean the angle between two rays emerging from the origin of coordinates and passing through the cuspidal points [4] on the wave front).

Also, formulas (6) enable us to determine the dimensionless ray velocities of propagation of quasilongitudinal and quasitransverse waves in a piezoactive cubically anisotropic medium:

$$g_m = \sqrt{(x_1^{(m)})^2 + (x_2^{(m)})^2 + (x_3^{(m)})^2}\,/t\,. \tag{7}$$

Let us investigate the influence of the piezoeffect on the change in the ray velocities of elastic waves, using bismuth germanate as an example. Figure 4 gives the dependences of the $g_m/\tilde{g}_m$ ratios for this material for the waves propagating in the plane $x_1' = 0$ ($\tilde{g}_m$ in the ray velocity of propagation of the elastic wave without allowance for the piezoelectric effect, i.e., for the coefficient $K = 0$).

From Figs. 2 and 4 it is clear that the piezoactivity of the quasilongitudinal wave propagating in bismuth germanate is low, since the maximum increase in the velocities v_1 and g_1 in the plane $x_1' = 0$ amounts to 1.75 and 0.2% respectively (in the coordinate planes, this wave is not piezoelectrically active). The influence of the piezoelectric effect on the change in the velocities of propagation of the quasitransverse waves in the plane $x_1' = 0$ is larger; thus, the maximum increases in the velocities g_2 and g_3 amount to 3.75 and 6% respectively as compared to $\tilde{g}_2$ and $\tilde{g}_3$.

In closing, we note that a comparative analysis of the velocity ratios g_m/v_m shows that we have $g_m/v_m \geq 1$ for the corresponding values of the angle of inclination of the normal to the characteristic surface.

This work was carried out with support from the Belarusian Republic Foundation for Basic Research (project No. F03M-171).

NOTATION

A_1, A_2, and A_4, elasticity constants of the cubically anisotropic medium; $a = A_1/A_4$; $b = A_2/A_4$; $c_2 = \sqrt{A_4/\rho}$, velocity of propagation of the transverse wave; e, piezoelectric modulus; $K = 4e^2/\varepsilon^s A_4$; $n_i = p_i/g$, direction cosines of the normal to the characteristic surface; $v = V/c_2$, dimensionless velocity of propagation of the discontinuity surface; $V = -p_0/g$; ε^s, permittivity; $\tau_1 = p_1^2 + p_2^2 + p_3^2$; $\tau_2 = p_1^2 p_2^2 + p_2^2 p_3^2 + p_1^2 p_3^2$; $\tau_3 = p_1^2 p_2^2 p_3^2$; $\tilde{\tau}_1 = n_1^2 + n_2^2 + n_3^2 = 1$; $\tilde{\tau}_2 = n_1^2 n_2^2 + n_2^2 n_3^2 + n_1^2 n_3^2$; $\tilde{\tau}_3 = n_1^2 n_2^2 n_3^2$.

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